Irrationality

Metric Structures and Quantified Space

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Introduction

To avoid any misunderstanding regarding the title of this thesis I will explain what I mean by the term irrationality. It is necessary to make a distinction between how this term is used in the field of music and in mathematics. In music it has sort of grown out of the term irregular, i.e. time signatures involving 5, 7, etc. When further step was taken into more complex time signatures, which involved irregularity in the denominator of the time signature, the term irrational was adopted. It is by no means the same complexity as for the mathematic version of irrational numbers, like the number π.

Irrational meters or so called fractional time signatures are terms used to indicate non-binary meters, i.e. a division of the whole note (semibreve) with numbers as denominators which are not in the power of 2, such as 4/10, 6/14 etc. These bars have often enigmatic cloud around them. They are not used that frequently because they are considered too difficult to deal with by performers. Though today you can find a slow growing usage of these kind of time signatures in solo pieces and even in some orchestra pieces.

When I started to compose, I struggled to write down things which didn't fit into our conventional notation system. I was not at all aware of the term "irrational meter". Through improvisation I encountered incomplete triplets and quintuplets, which didn't fit on paper. My first score in which I made use of "incompleteness" to some degree was clumsy, as it was changing tempos nearly every bar. Later I discovered that fractional time signatures or irrational meters were precisely the elements needed to notate what I had in mind then. Out of curiosity, and because it became important to me, I set forth to research how irrational meter has been used and is being used, in order to enhance my understanding of it.

When exploring the history of fractional time signatures or irrational meters one will encounter Henry Cowell, who wanted to establish a relationship between harmony and rhythm. Cowell explored the proportions in the overtone series and used them to create new durational note values, which had same proportions as the tones in the overtone series. The new durational note values gave birth to new time signatures or meters, which contained these new notes. This influenced many composers in the 20th century, one of them being the experimental composer Dieter Schnebel. The scores of Schnebel later influenced Brian Ferneyhough who explored this phenomenon further. Ferneyhough and his students have used irrational meter structures, in their own way with their own aesthetics, since. Other composers, often of different aesthetic convictions, like Thomas Adès, Jo Kondo and others, also made use of irrational meters and will be covered briefly in this thesis for a broader perspective on the phenomenon.

The main question and goal of this research is to explore the use of irrational meter, i.e. the ways in which it has been used, as a compositional tool. Also, I want to explore why it is used in a particular way and if this practice has grown out from a certain aesthetic point.

Further questions concern the teaching of irrational meters. The use of them often results in “impossible-” or unperformable scores, in the opinion of many. Can this possibly we avoided by offering adequate education? Or, is the material beyond human capacity, per se? Why are the recent refinements of our rhythmical notation not reflected in music education? Is this a huge step, or a minor alteration of established training programs in the field of rhythm? I will suggest some reforms, as a provisional answer to these questions.

1. Encounter and Definition

1.1 The Phenomenon

As explained in the introduction the terms *irrational meter* or *fractional time signature* divide the pulse of the semibreve (whole note) with numbers (in the denominator), which are not in the power of 2 e.g. 4/10 or 7/15 etc. So, we face a slightly more complicated rhythmical phenomenon than those represented by conventional time signatures (4/4 or 3/16 etc.). What these terms “irrational” or “fractional” point at is a new and different approach to tempo changes, i.e. proportional changes (proportional, because the only paradigm is the *last* pulse and not a single, unifying one). Let us explore this further. Usually when composers write a higher or lower pulse, which is not dividable by 2 (without fractions), without changing the tempo they do stay within the regular pulse system (**Ex.1.1**). [1]

Ex.1.1

This yields tuplets, which divide the established pulse into 3, 5, 6, 7 beats etc. One starts and ends on a beat of the main pulse (at least in the mind of the performer).[2] What an irrational time signature indicates is a proportional leap, a change of tempo conforming to the speed of a “tuplet”, with the possibility to leap out again at any point within the “tuplet”, thereby, suggesting an elimination of the term tuplet. Let´s say we are in 2/4 (**Ex. 1.2**) and then there is a jump to 4/20.

Ex.1.2

What happens here is that the music goes from the pulse of a quarter note into a pulse of a twentieth note, which is normally called a quintuplet time but since quintuplets

[1] Pulse system stands here for pulses within a given time signature.

[2] Of course tuplets don’t always start and end on a beat but here the idea is that they are a whole unit in themselves i.e. quintuplet is always five notes.

have 20 notes in the semibreve (4*5=20)[3], and here there are only four notes instead of five, these notes are better called “twentieth” notes. This could also be represented by tempo changes: quarter note ♩=***40*** → twentieth note ♪=***200***, but if the metrical structure of a piece uses these changes up to a certain amount it becomes easier, more practical and clear to use irrational meter.[4] This kind of music ceases (or refuses) to move within a regular pulse system, which has occasionally different kinds of tuplets. Instead it has individual planes of tempos, which are related by certain proportions, presented by precise divisions of the semibreve, and can be quantified according to any rule.

Throughout history of musical notation, in the sense of time organization, there seems to be a progression towards more complex fractional division of the semibreve.[5] In the beginning of the 20th century there appeared more usage of time signatures such as 3/8 or 5/16, but, here the division is in the power of 2.[6] The cause of this is simply that it is easier to divide by 2 (rhythmically) than 3 or 5. This was, though, an important step in history and created enough difficulties with performance and nobody thought of going any deeper into the division, at that time, except Henry Cowell.

1.2 Henry Cowell

“… a note occupying a third the time of a whole note a ‘half note triplet,’ why not refer to it as a third note”. [7] Henry Cowell wrote this in his book *New Musical Resources*, which was published 1930. Quite ahead of his time, we could say, he implies different divisions of the semibreve (pointing towards new time-signatures), i.e. with third notes etc., and that we should reconsider the term tuplets in a more structural way.

Cowell’s approach towards fractional time signatures is quite unique. He wanted to create a new relationship between rhythm and harmony i.e. “harmony and counterpoint in rhythm, which have an exact relationship to tonal harmony and

[3] Quarter note means four equal notes in the semibreve (whole note), quintuplet here is five notes in the quarter, therefore, 4*5.
[4] Just as we don’t change the tempo for 16th note bar why do it for 20th note bar.
[5] One could say that it’s linked with Adornos theory of rationalization.
[6] E.g. music of Schoenberg and Stravinsky. 8th notes and 16th notes are divisions of power 2.
[7] Cowell, 1996, p.54.

counterpoint."[8] He looked into the proportions of the overtone series and constructed an equivalent series for rhythm, since pitches are of the same nature as pulse, only at higher frequency, he could easily use the same proportion for rhythm and harmony. Further on he created a rhythm scale, which took into account the overall ratios between the intervals of the chromatic scale (**Ex. 1.3**). Here, every interval has a ratio and a durational note value in its relation to the semibreve.

Ex.1.3

Intervals from C.	Ratios from C.	Tones of Chromatic scale	Corresponding Time Value
		C	4^{th} note
Aug. unison	14:15	C♯	$7/30^{ths}$ note
Major second	8:9	D	$2/9^{ths}$ note
Minor third	5:6	E♭	$5/24^{ths}$ note
Major third	4:5	E	5^{th} note
Fourth	3:4	F	$3/16^{ths}$ note
Dim. Fifth	5:7	G♭	$5/28^{ths}$ note
Fifth	2:3	G	6^{th} note
Minor sixth	5:8	A♭	$5/32^{nds}$ note
Major sixth	3:5	A	$3/20^{ths}$ note
Minor seventh	4:7	B♭	7^{th} note
Major seventh	8:15	B	$215^{\text{ths note}}$
Octave	1:2	C	8^{th} note

[9]

This table inspired, among others, Nancarrow to write his proportional canons for player piano. The table (**Ex.1.3**) shows the change of frequency, written in the form of durational note values and proportions of intervals within one octave. If "C" is a quarter note, a "G" is a 6^{th} note and equivalent ratio between those two speeds is 2:3.

"Our system of notation is incapable of representing any except the most primary divisions of the whole note" (Cowell 1996, p.56) Cowell stated and was concerned with making new shapes for note- heads which would represent different lengths or a different division of the semibreve in themselves, mainly those presented on the right side column in the table above (**Ex.1.3)**. In the explanations of the piece

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[8] Cowell 1996, p.46.

[9] This table is a reconstruction of a table from Cowell's book *New Musical Resources* p.101. For reasons explained in NMR p.99 the C is a quarter-note.

Fabric (1917), where he continually uses those note-heads, Cowell gives an outline of these note-heads and their durational values (see appendix).

With all these new notes, with different time values, he created new material for time structures and metrical structures with time signatures which divide the semibreve in a more fractional way, "…new meters could be made by using the new kinds of notes suggested in the time-schemes, 2/3, 3/5, etc."[10]

1.3 Roundup

We can now see that new note values, e.g. 20th note, which give rise to new time-signatures, are only a natural[11] progression towards further development of music, or unavoidable, just as chromaticism and later microtonalism in the pitch area of music, the same is represented on a lower frequency rate, mainly, in the rhythmical dimension of pulse. We see also a limitation of our established notation system in regards to pulse organisation. In the next chapter we will see, through analysis, how this phenomenon of irrational meter or fractional time signatures has been developed further in the 20th century to considerable complexity.

One aspect of irrational meter is that a meter can now have the function of changing the tempo, in some sense. When a piece has different irrational meters it is in fact working on different levels of tempo, but these tempos are proportionally related and that gives rise to the usage of irrational meters. Also, irrational meter expands the material of composition, it gives extra dimension towards pulse, tempo and rhythm.

The terms I've been using, *irrational* and *fractional* time signature, are by no mean clear enough. Irrational, in relation to numbers, means numbers that cannot be represented as simple fractions.[12] We are hardly dealing here with time signature with numbers like π. Fractional is not good either since any time signature which is not whole, i.e. like 4/4, is fractional, 3/16 is as much a fractional time signature as 7/15. But since I have not been able to find better terms I will use (as a matter of taste) irrational meters.

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[10] Cowell 1996, p.66.

[11] One can argue if it is at all natural but it is at least the fixation of the human mind which gives these results.

[12] 3/7 is a simple fraction $7/\sqrt{2}$ would be irrational

2. Analysis

In this chapter I will look at few pieces and texts by selected composers from the 20th century who applied irrational meters in their work. The discussion will vary between composers as the material, in relation to the subject irrational meter, differs from each of them. This has mainly to do with sources, i.e. text-sources or scores-sources. Our first subject, Dieter Schnebel, will be treated sort of systematically and generally since he is mainly important for making an important step for the other composers. Following Schnebel we will look at Brian Ferneyhough which has written scores and text concerning irrational meter. Claus-Steffen Mahnkopf will be treated mainly by analysis of two of his pieces, but Frank Cox has written about his work in regard to our subject and will be treated accordingly. Last I will touch upon Thomas Adés and Jo Kondo for their different approaches towards irrationality.

2.1 Dieter Schnebel

Dieter Schnebel in the early 1950's wrote some pieces with striking metrical challenge. These pieces he called *Versuche.*[13] They were made up of four pieces. There, we can see the result of Henry Cowell's thought in practice.

In the performance notes of the second piece (*Stücke for strings (string quartet)*) Schnebel states:

> "In the time-signatures, the 6ths and the 5ths indicate ♩ divided into six or five parts; 18ths and 20ths : the same values sub-divided three or four times."

Here one could misunderstand that the 6ths and the 5ths indicate a division of the quarter note, but he clearly means that in those specific bars, like 4/6, he uses the quarter note with a transformed durational value, i.e. a quarter note within a 4/6 bar is actually a 6th note (semibreve divided 5 or 6 times would be correct). Here, Schnebel does not deal with the problem of note heads, i.e. a 6th note should have a different note-head from a quarter note for clearer distinction[14], or rather he finds a solution; he merely uses the quarter-, 8th-, 16th notes etc., and lets the time signature define, or

[13] Schnebel 1973
[14] Like Henry Cowell was trying to establish. (See appendix A)

transform, its duration. In this way a half note can be transformed to a 3rd note, a quarter note to a 5th note, 6th note and a 7th note and then we have the conventional 8th note which covers up to the 16th and so forth. This creates a new meaning of our notes, they have now a direct relationship with the time signature or the meter they are in.

This piece, *Stücke,* is constructed of five very short movements. In the first movement of *Stücke* the metrical structure is the following:

5/8, 3/8, **4/6**, **4/10**, 8/16, 8/16, **8/10**, **8/6**.

Second movement:

3/4, 5/8, **4/3**, **8/5**, 5/4, 3/8, 12/8.

Third movement:

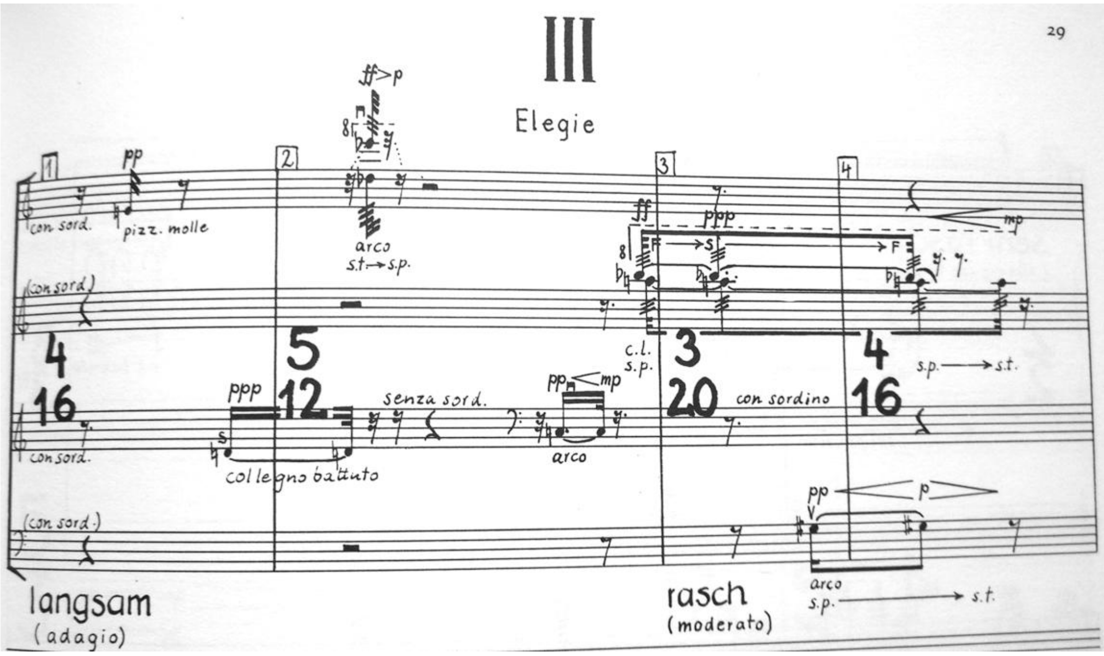
4/16, **5/12**, **3/20**, 4/16, **8/20**, **8/12**.

Fourth movement:

2/2*4, **3/3**, **5/5**, 4/4, 8/8*3.

Fifth movement:

5/3, **3/12**, **4/5**, **2/6**, **5/24**, **19/15**, **19/18**, **18/36**, 4/2.



Ex.2.1: Beginning of the third movement of *Stücke*

We see that irrational time signatures are clearly among the main factors of this composition. These structures give us an image of how the underlying pulse is constantly moving up and down, fast and slow, in a quite precise manner. On top of these metrical complexities Schnebel freely uses ritardando and accelerando for even further enrichment. One could say that here the usage of irrational meter is to control the tempo in a precise manner, mainly to create a time/space/durational structure and not so much for rhythmical complexity. But, when you look at the musical material employed within these bars, on the contrary, there is rhythmical complexity *because* of the structure (though one could of course imagine much more rhythmical[15] material within the bars). The use here, though, is rhythmical in a different way, because even with a very simple notation attached to this structure you have difficulties with durational details of notes. This is very often the case in this piece, i.e. simple notation/rhythm in a complex environment (see **Ex.2.1**). The weight of a single note increases comparatively with the durational preciseness of the bar structure.

Occasionally, Schnebel uses a common division or phrasing in the numerator (upper number) of the time-signature, e.g. in the first movement we see 8-8-8 in the numerator but 16-10-6 in the denominator (the lower number) and in the fifth movement we see 19-19 (numerator) over 15-18 (denominator), but the material within the bars are only occasionally emphasising the change of meter. The music is very much an extension of the "pointalism" and "klangfarben-melodi" of Webern (Schnebel also employs serial techniques), in these manners he finds a way to give more weight and tension to a single note (as mentioned above) with irrational meters. Actually the result is a simplification of notation of a desired rhythmically complex music. In the third movement we see him actually playing with numbers: 3/4 becomes 4/3 and 5/8 becomes 8/5 and then the others combinations 5/4 and 3/8 becomes 12/8 (4+8=12 and 5+3=8). In this work Schnebel only uses 5th and 6th notes or a division of them and subdivisions, e.g. 15th (3*5) and 18th (3*6) notes.

Dieter Schnebel usage of irrational meter is revolutionary. It set the standards in irrational meter usage for the coming generations.

2.2 Brian Ferneyhough

Brian Ferneyhough is probably the person who transformed the way we think about, and how we use, meters and especially irrational meters. Most of his compositions are thoroughly organized in time, by meters. It's then no wonder that he

[15] "Rhythmical" means here a motor-like rhythm (e.g. ostinatos, motifs, pulse attached material, repetitions)

took up this addition to our conventional meter system. He has applied irrational meters in many ways, and has written a great deal about their qualities and values. The first piece, which acquainted Ferneyhough with irrational meters was Schnebel´s *Versuche.*[16] Ferneyhough makes use of the same approach towards notation, i.e. he does not employ new note-heads. However, he developed his ideas, about the use and adaptation of irrational meters, much further. In an essay called *Duration and Rhythm as Compositional Resources (1989)* Ferneyhough says about his general rhythmic treatment:

> "In my own music I have concentrated on the issue of interrelating iterative rhythm and metric structure through ratio relationships which are individually quantifiable, and thus open to interaction in many diverse and complex ways."[17]

An overall analysis of his use of irrational meters would be based on the following types of meters, i.e. ratio or proportional relationship types:

a) Triplet proportional relationships (denominator: 3, 6, 12, 24 etc.),
b) Quintuplet proportional relationships (denominator: 5, 10, 20, 40 etc.)
c) Septuplet proportional relationships (denominator: 7, 14, 28, 56, etc.).

In his work you don't find more complex meters than these.

In order to understand how Ferneyhough uses irrational meters one has to understand his methods of meter/measure construction with regard to musical material applied to such structures and how he aesthetically perceives a measure or a bar.

> "…a measure is not primarily a unit of emphasis, of agogic priorities, but a space, serving to delimit the field of operations or presence of specific sound qualities, of musical processes."[18]

Ferneyhough perceives a bar as a space, which delimits operations. This is important - to grasp, that a space opens up possibilities for work, much in the same way as a piece of paper enables the painter to proceed and a character enables the writer to write. A

[16] Interview ETE/BF in appendix
[17] Ferneyhough 1996, Collected Writings, p.51-52.
[18] Ferneyhough 1996, Collected Writings, p.53.

durational space provides the freedom to work and at the same time hints at more such spaces with different sizes, to develop a musical process. This is very much the way Ferneyhough aesthetically views the compositional process. About his compositional process and the pressure needed to compose he says:

> "Forcing the mass of creative volition against and through prestructured grids or sieves ensures this pressure and breaks the amorphous mass up into various types of activity or function."[19]

Prestructured grids are here obviously the metric structure. Further on the "space" Ferneyhough continues:

> "The consistency of iterative impulses serves primarily to set off the limits, operative boundaries between one such space and another."[20]

Here we observe that a measure (a bar) and the impulses within a measure, or superimposed impulses on a measure-space-structure, are means to limit the material of a particular structure and their internal connections, sort of a focus point on the working platform. The measure delimits a field of operations and the impulses limits those operational processes, sort of a fine-tunement of the material. In this view it becomes very important for the formal organization which meters you work with, since they have double functions, and the consistency of impulse occurrences. First you define the durational space, which delimits certain field of functions or processes and then you define its internal clocks, which limits certain operations or activity types attached to it. A 5/8 bar has certain duration and a certain impulse structure, a 5/24 bar has the same impulse structure but another duration.[21]

> "Expressions of ratio relationships and proportionally perceptually related structures are, in essence, expressed by means of different categories of perceptual mechanism."[22]

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19 Bons 1990, p. 15.
20 Ferneyhough 1996, Collected Writings (p. 52)
21 Within same tempo of course
22 Ferneyhough 1996

Subdivisional impulse, like 7:5, can relate those two measures (the 5/8 and 5/24) in a refreshing way and gain a new, *density-expression*[23] relationship, which retains the same relationship in a new category of perception. Density expression is a term which is used for the amount of activity within a measure or a certain area, e.g. you can increase the density-expression-level by superimposing a subdivisional impulses on top of a measure (e.g. 7:5 over a 5/8 bar).

The above gives us a useful picture of how Ferneyhough's world of rhythm and meter is constructed and thoroughly thought of. Further on he says about his work in relation to meter:

> "An immediate consequence in my own work has been the basing of compositional-formal structures on cycles of recurring (and thereby progressively transforming) measure durations."[24]

The progressive transformation of measure durations is a key element in Ferneyhough's composition technique. For an example of this we will now look at the first piece in which he made use of irrational meters, *Lemma Icon Epigram* for solo piano.[25]

2.2.1 *Lemma Icon Epigram*

To avoid any misunderstanding we first have to clarify one point in relation to this piece. In the performance notes of the piece *Lemma-Icon-Epigram* (1981) Brian Ferneyhough mentions unconventional bar-lengths:

> "Unconventional bar-length notation is encountered in the later sections of the piece (2/10, 3/12 etc.). In each case, the principle applicable to the derivation of more customary lengths (understood as equal subdivisions of a breve) is maintained. For example, 2/10 signifies two beats to a bar, each being equal to one-tenth of a breve. All such passages in this piece have an internal beat faster than the original quaver beat."[26]

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23 Ferneyhough 1996, Collected Writings, p.53
24 Ferneyhough 1996
25 Interview ETE/BF see appendix B
26 Ferneyhough 1981.

Here we find a confusing statement about the note values. He says that a 2/10 bar is equal to one-tenth of a *breve*. This is clearly a mistake and should be a *semibreve.* This is important to clarify in order to know exactly the durations of notes, since some composers have written a 5/3 bar, which then indicates (wrongly) a triplet relationship equal to five 8th-triplets.[27]

We can now proceed, without confusion, to see the metrical structure of the middle part, of this very piece (*Lemma-Icon-Epigram*), which contains irrational time signatures.

Lemma Icon Epigram is, like the title suggests, divided into three parts. Though directly connected to each other, the parts have quite different processes. In the middle section (*Icon*), however, we see an overall usage of irrational meter and a quite specific usage indeed. It is interesting to read the description Ferneyhough gives of this section:

> "The idea here was a temporal sun moving across an irregular but fixed landscape, with objects placed in it. The landscape is of course the bar structure; the temporal sun is the ticking (if you want to be over-literal for a moment) of these groups that gradually emerge, and the objects are the chords, which scrunched up and expanded both in length (growing and getting shorter) and in density (register). … All these things together produce the feeling of an intensely but mysteriously temporal phenomenon."[28]

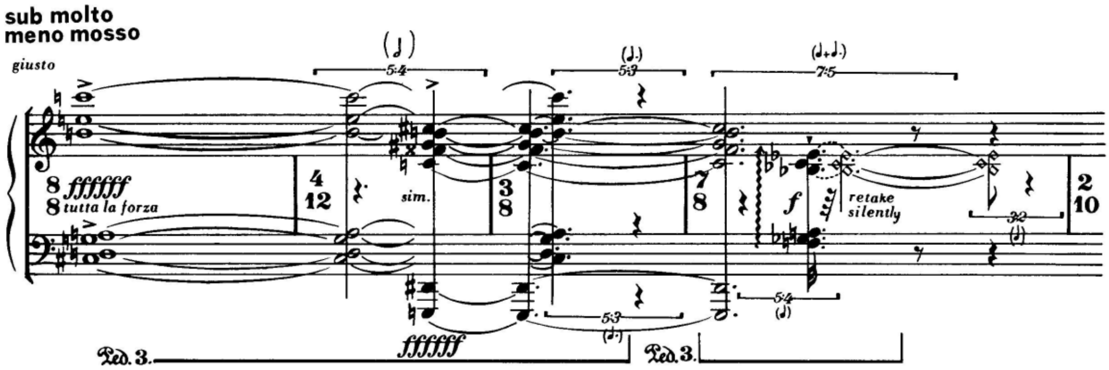
Recurrent cycles are here a dominating theme of the construction. And through the "moving sun" every cycle is transformed in duration by applying the usage of irrational time signatures, only of type **a** and **b** are used, though.[29] It is as if a different perspective point is being altered gradually. The basic scheme for the first cycle is the following (**Ex.2.2, 2.3)**:

$$\underbrace{\tfrac{8}{8}\quad\tfrac{4}{12}}\quad\underbrace{\tfrac{3}{8}\quad\tfrac{7}{8}\quad\tfrac{2}{10}}\quad\underbrace{\tfrac{5}{8}\quad\tfrac{4}{8}\quad\tfrac{6}{8}\quad\tfrac{1}{12}}$$ [30]

Ex.2.2

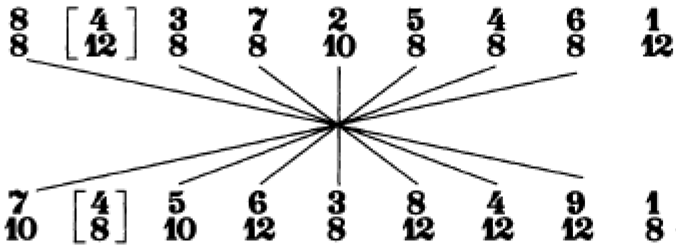
27 Which is then equal to 5/12 bar.
28 Toop 1990, p.80
29 See chapter 2.2 (beginning)
30 Toop 1990, p.80-81

We can see that, for the numerator, Ferneyhough uses numbers from 1 to 8, inclusive, which are distributed non-chronologically. Here the irrational meters could be viewed as an element of deformation of the underlying pulse. The transformation process for



Ex.2.3: Beginning of the first cycle

the following cycles involves retrograding, adding and the interchange of rational (the eighth bars) and irrational values (the tenth and twelfth bars).



[31]

Ex.2.4

In **Ex.2.4** we see the transformation to the second cycle. Ferneyhough applies retrogradation, adds one to the numerator, and interchanges the "eighth" bars with "tenth" and "twelfth" bars (the bars with brackets are filtered out of the retrograding process). There are seven cycles of transformation. In the end (in the 6th cycle (**Ex.2.5, 2.6**)) the first cycle returns with the same impulse structure but different durations (i.e. the denominators). All the "eighth" bars from the first cycle have now become "tenth" or "twelfth" bars, "tenth" bars become "eighth" bars and the "twelfth" bar becomes a "24th" bar.

$\frac{8}{12}$ $\frac{3}{12}$ $\frac{7}{12}$ $\frac{2}{8}$ $\frac{5}{10}$ $\frac{4}{10}$ $\left[\frac{3}{8}\right]$ $\frac{6}{10}$ $\frac{1}{24}$ [32]

Ex.2.5

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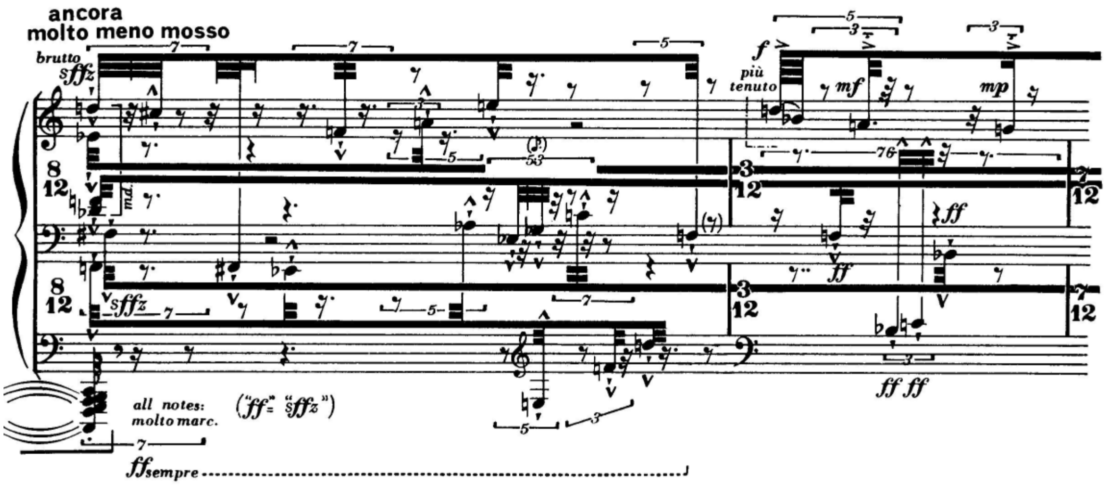
[31] Toop 1990
[32] Toop 1990

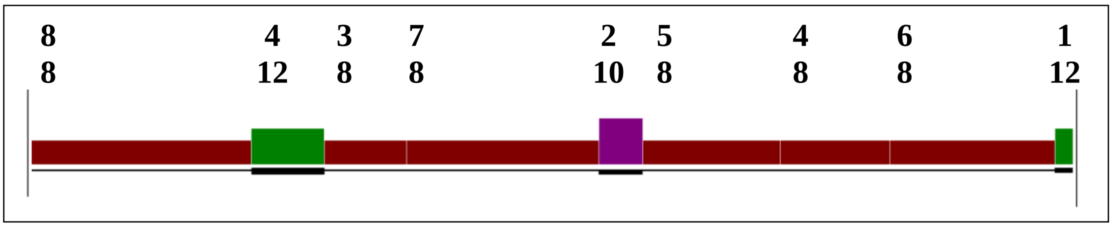


Ex.2.6: Beginning of the 6th cycle

On this space structure (landscape), these cycles, Ferneyhough superimposes a number of impulse groups (which are the "objects within the landscape (see quote above)), which are spread over these cycles.[33] Each impulse group has fixed intervals (i.e. of silences) between its attacks, which is out of phase with the meter structure. We see a simple start of this in the first cycle in **Ex. 2.3**, but more complex one in the 6th cycle in **Ex. 2.6**. This resembles the renaissance idea of color and talea but in this case there are actually two taleas: the impulse group and the bars.

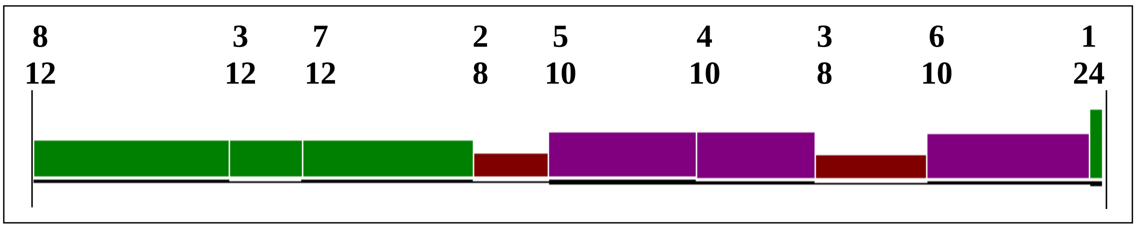
It is useful to make an image or a visual perception of the spatial proportions in the "landscape" of the metric structure. If we only take the first cycle then the landscape would look like this:

Ex.2.7



For contrast let us look at the 6th cycle to see the transformation:

Ex. 2.8



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33 See appendix C, Impulse Scheme

In these two examples (**Ex. 2.7** and **2.8**) we can see that irrational meters slowly start to dominate the “landscape”. We can also say that we have now a different perspective on the same landscape, things are differently closer and further away than before. Also, one could say, the function of the rational and the irrational meters have been switched, i.e. the rational meters serve now as deforming elements. The material within is differently stretched. It can be discussed if this is at all perceivable in the performance but it is obviously perceivable structurally and abstractly.

This sort of treatment of irrational meter triggers the imagination. This dense structural thinking is by no means limiting Brian Ferneyhough’s compositions, he mainly uses these structures to be able to work “freely” within them, that is his own individual way, though, highly intellectual and abstracted. It is no wonder that Brian Ferneyhough influenced a lot of his students towards this direction of thinking about irrational meter structures. Many of his students have taken up usage of irrational meters in their own way or in a similar way, and even up to a certain saturated point.

2.3 Claus-Steffen Mahnkopf

Claus-Steffen Mahnkopf studied under Brian Ferneyhough and Klaus Huber in Freiburg and one can see that from them he arrived at his treatment of irrational meters. Mahnkopf, like Ferneyhough, is a complexity composer. He has explored the use of irrational meter in his own way, and though we can find similarities with Ferneyhough’s music, the thought and the organisational aspects of employing irrational meters is different.

To gain inside into a use of irrational meter different from Ferneyhough´s or Schnebel´s, let us look briefly at two of Mahnkopf’s pieces for piano, *Rhizom*[34] and *5 Kleine Lakunaritäten*[35], where he applies irrational meters. It is interesting to see that the usage differs quite a lot between those two pieces.

2.3.1 *Rhizom*

In *Rhizom* the metric structure is quite a dominating factor and very related to the musical thinking, e.g. of individual lines or voices within the music, i.e. the

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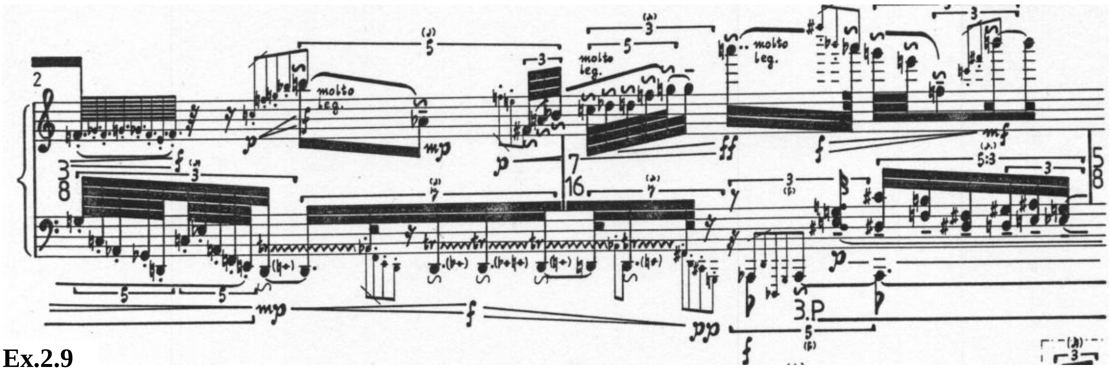
34 Mahnkopf 1989
35 Mahnkopf 1995

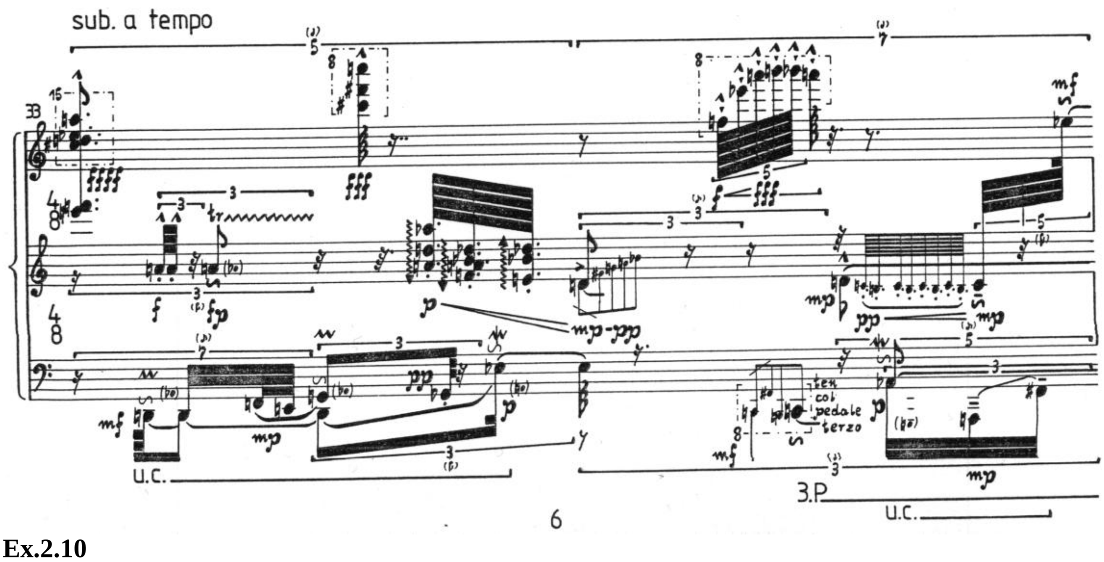
polyphony involved. There is a gradual process going on towards something that we could call a rhythmical or metrical chromaticism, a certain growth in temporal tension between the voices.

In the beginning of the piece we encounter only conventional bars, i.e. with different 8^{ths} and 16^{ths} divisions, with dense material attached to it (**Ex.2.9**).[36]



Ex.2.9

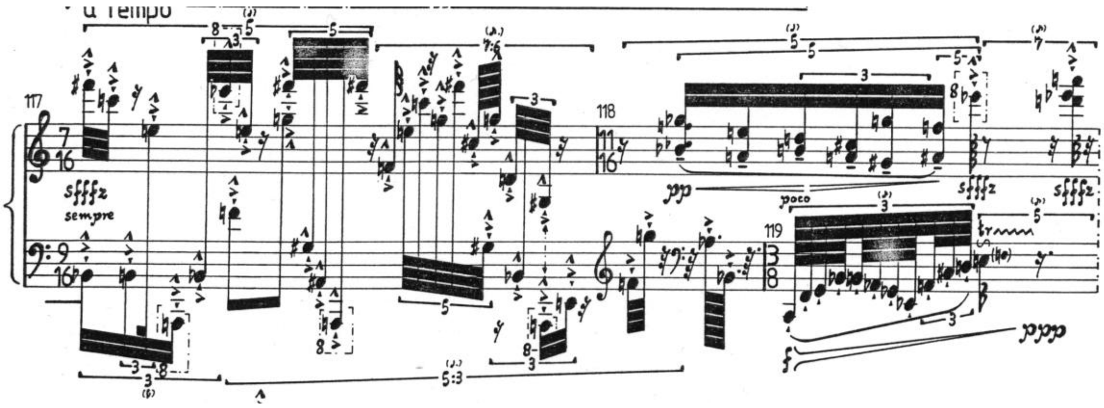
Until bar 24 there is only a complex counterpoint between two voices present. In bar 24 a third voice is added, first only as pedal but in bar 29 this voice becomes more important. The notation emphasizes this importance with a three stave system (**Ex.2.10**). This complex counterpoint eventually explodes in density (chords) and the voices become again only two in bar 77.



Ex.2.10

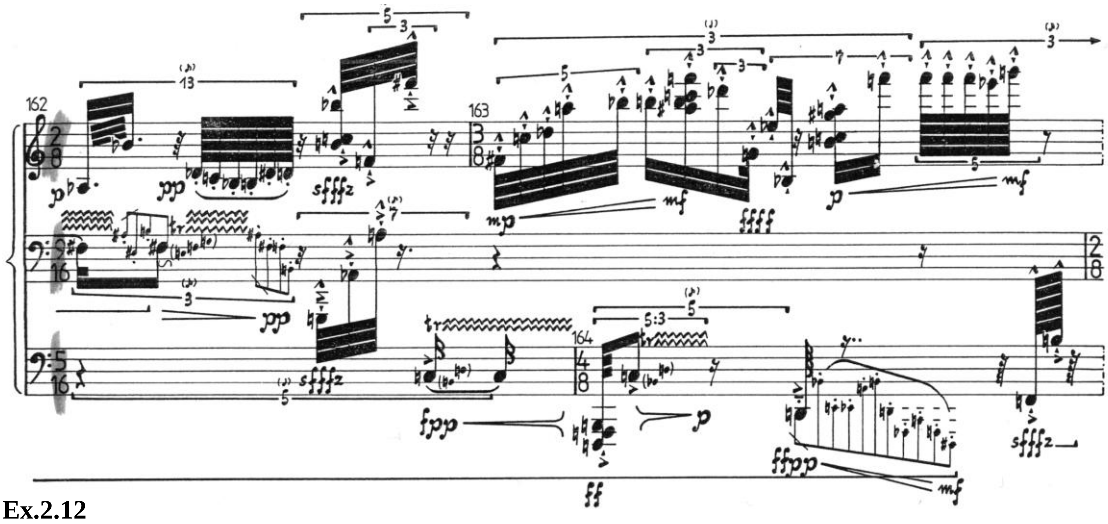
[36] I don't go into details concerning other material than what concerns the metrical structure, but since the usage of irrational meter here is very related to the direction of lines or voices I have to mention a few things.

At bar 117, which we can see in **Ex. 2.11**, we see an important change in the metric structure, there starts a polymetrical division between the two voices. These two voices have now two different impulse structures and direction. In bar 162, see **Ex.**



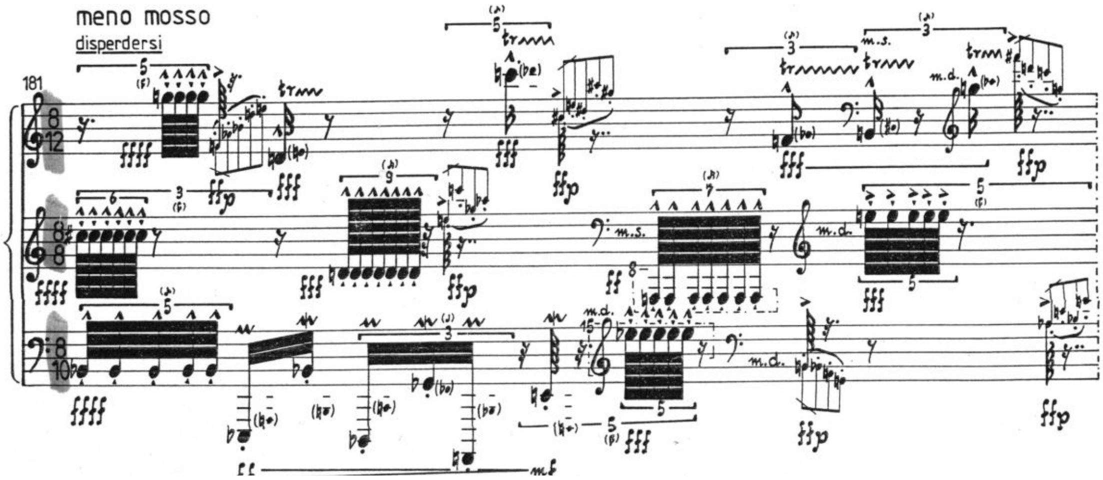
Ex.2.11

2.12, the third voice enters again and has now also its own metric structure. The polymetrical structure has now three layers. For **Ex. 2.12** we can say that those three



Ex.2.12

voices have sort of gained their individuality, but not completely, since they are all attached to a common underlying pulse (i.e. still all in the speed of a division in the power of 2), though of course they have different subdivisional phrasings etc. What happens now in the piece, approaching its end, is that those three voices break their tension into even more tension (**Ex.2.13**). Irrational meters are now employed in order to let those three voices have their own individual pulse structure (polypulses) in precise proportions to each other. In bar 180, right before the break, the three layered polymetrical structure comes together in a common 13/16 bar only to finally break apart in the next bar into three layers again (**Ex.2.13**), only now we see an *irrational* polymetrical structure: 12th notes against 8th notes against 10th notes. First they have a common basic impulse structure, i.e. they all share the eight in the numerator of the



Ex.2.13 Bar 181, irrational polymetrical structure.

bars, but these bars have different durations, and later they lose their connection with this common factor. The three voices interplay in a fragmented way, in their own meter structure, organised in a precise manner with irrational meters, towards the end of the piece.

The usage of irrational meter in this piece is the goal of its intension, unseperatable from the idea of the piece and the motion of the energy. We could say that those three voices were gradually chromatically, in a rhythmical or palpitational sense, altered. Just like if we would chromatically alter a note in a chord and then immediately change the relationship between the notes of that chord.

2.3.2 *5 kleine Lakunaritäten*

In the above piece, Rhizom, we only saw the usage of irrational meter type **a** and **b**.[37] In this piece, *5 kleine Lakunaritäten,* we see quite a radical change in the use of irrational meter types. We have to invent new types to cover what is being used in this piece. Here Mahnkopf makes a similar tempo scale as Henry Cowell constructed, only he goes even further than Cowell thought wise. Throughout the whole of those five small pieces there is a constant usage of irrational meters. Bars constructed of 8th notes towards 15th notes are used. We have eight steps in this "tempo scale", e.g. in the first piece the following tempo scale is used:

8th notes-9th notes-10th notes-11th notes-12th notes-13th notes-14th notes-15th notes

37 See chapter 2.2

The tempos for this scale is given in the performance notes as this:

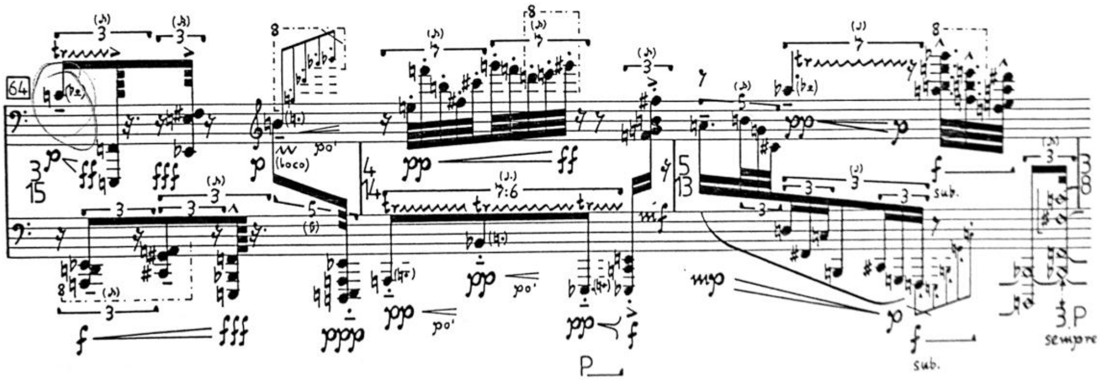
43	48.375	53.75	59.125	64.5	69.875	75.25	80.625

We can speculate if this is at all humanly possible to accomplish to any accurate degree. The method Mahnkopf uses in this piece in order to connect these irrational meters is with common numerators or a limited types of division. To use a common numerator (as we saw also in Schnebel) in order to change the speed is very close to how classical Indian music (mainly carnatic music) deals with movement from certain pulse or subdivision to another.[38] Classical Indian music calls this treatment *jathi-relationship* and it has to do with phrasing. E.g. jathi 5 is a phrase in five[39], which could be played in any speed (quintuplet, septuplet etc.), i.e. 5/8 bar could be expressed as 5/11 bar, while keeping the jathi 5 relationship. In order to move to a different speed, according to carnatic music rules, you should keep the same phrasings, in this case the numerator (5), in order to maintain a relationship and for a fluent establishment of the new pulse. It seems that Mahnkopf here uses a related principle, or an extension of this Indian thought. In the first piece of *5 kleine Lakunaritäten* we encounter only bars with numerator as 6 or 3 (a clear relationship) but, as mentioned before, the denominator is constantly changing. In the second piece we see only 4 or 5 as numerators and we see this sort of progressions: 5/8 – 5/10 – 5/9 – 4/9. This is very related to the *jathi* concept, which also changes the numerator once the new speed[40] has been established, i.e. the 5/9 > 4/9 (though they take more time for the establishment in carnatic music). Also, we can see that this is exactly the same thought as in elementary modulation in the tonal harmony system, where you have a pivot chord to move to a different key. In the third piece we see a continuation of the 4 and 5 (jathi) relationships and in the fourth piece, again we have 3 and 6 relationships only. The last piece has more challenge to it. There, he combines the two working methods previously used. So, we see bars with 3, 4, 5 and 6 in the numerator and only occasionally he applies the “jathi” method (**Ex.2.14)**. There is this sort of relationships though: 6/11 – 3/15, but that’s about it. The piece moves therefore towards more daring meter modulations (if we can use the harmony metaphor again),

[38] I owe my knowledge of South-Indian Carnatic music to the teachers of the course “Contemporary Music through Non- Western Techniques” at the Conservatory of Amsterdam.

[39] Five pulses of the same value.

[40] New speed is here the new denominator.



Ex.2.14: The last piece of the 5 kleine Lakunaritäten.

just like the tonal music evolved its harmonies this piece does that as well in a rhythmical sense.

It seems to me that in both of these pieces by Mahnkopf there is some connection to traditional harmony, only transposed to the field of meter. There is also some connection with the thoughts of Henry Cowell concerning harmony rhythms and tempo scales[41], much more than in the music of Brian Ferneyhough.

2.4 Frank Cox

Frank Cox is another student of Brian Ferneyhough who has made irrational meter into his main compositional material. He spend several years developing what he calls a *Non regular Metric rhythmic language*.[42] Frank Cox states that the drive behind developing this system or language was:

> "…a critique of the entire Western tradition that conceives of the hierarchical organization of meter and rhythm as inextricably bound up with regularities."[43]

Further about the goals he hopes or wants to achieve with this new language he says:

> "it attempts to transform this tradition from within by creating a dynamic metric-rhythmic hierarchy fundamentally based on non-regularities."[43]

41 see chapter 1

42 Cox 2004, p.90

43 Cox 2004

With resistance against any regular pulse, for more than a single bar, he makes use of irrational meters in order to establish this non- regular language. He criticises the way meter in general is being used in modern and contemporary music.

> "…there are countless cases of interesting- looking metrical structures which could in fact easily have been notated otherwise, their role mostly a matter of notational convenience, and not properly metric. There are also many cases of elaborately calculated metric hierarchies whose perceptive role is negligible…"[44]

Here he touches up on an important subject, i.e. the perceptive role of meter. In many pieces, which utilize complex metric structure and especially irrational meters, you actually seldom experience the different levels of tempo in an accurate way on the moment a new meter has arrived. The musical material, which these irrational bars contain, is so rhythmically complex in itself that changing the underlying pulse is hardly perceivable. Sometimes, like in the music of Brian Ferneyhough, there is another thought behind the bar structure, though, a certain abstraction, e.g. that the bar is considered as a structural durational space, which does not apply to this discussion in my opinion because then we are dealing with a different aesthetic point of departure. [45]

Frank Cox wants that a meter retains its original function and quality, he wants to renew the original aesthetic function of meter:

> "I have chosen to befriend beat and meter, as they are powerful structural units for rhythmic organization, and they contain a vital element of physicality, with the downbeat enacting a landing of bodily weight, with the rhythmic action within the meter conveying a rebound and lift before the next landing…"[46]

In this sense he is a traditionalist but with his obsession for non- regularity he innovates. One of the most important elements in his rhythmic language is his use of what he calls "*taleas*". A *talea* is a row of numbers, like 6-4-7-3-5. Out of this row he

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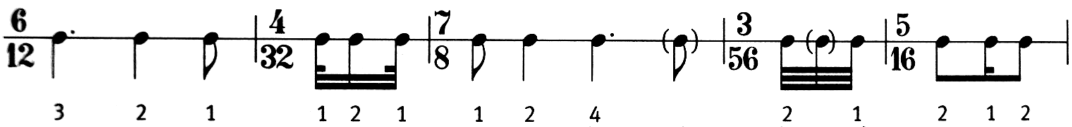
[44] Cox 2004
[45] See chapter about Ferneyhough 2.2
[46] Cox 2004, p.94

tries to create all his material through the means of transformation. He says, about how he can treat any of those *taleas*, the following:

> "…treated as energy shapes, curves of momentum, determinate quantities and relationships, intervallic proportions, metric/durational/speed proportions, motivic pool, and so on."[47]

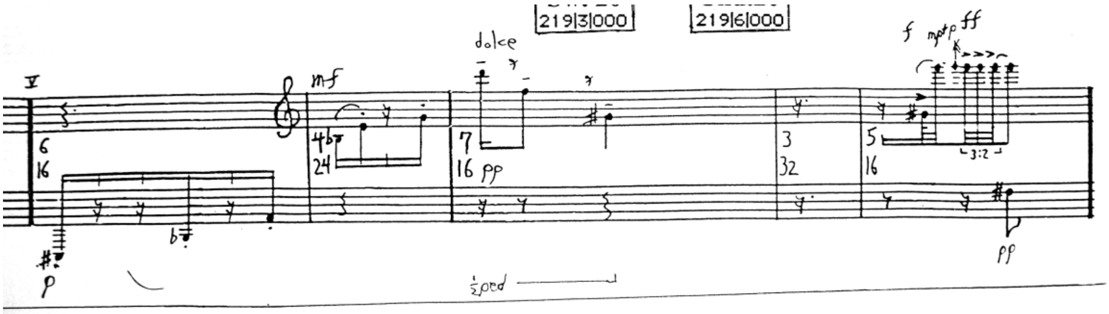
Let us look at one way he develops such a *talea* in relation to irrational meter. In the piece *Doubles* for piano and tape by Cox we see this "language" being realized. There he uses three of these *taleas* to generate all his material. Let us take the *talea* mentioned above, which he uses for the A section in this piece *Doubles*.
6-4-7-3-5 can be treated in many different ways. One way is to treat it as a series of proportions, i.e. 6:4, 4:7 etc. these numbers can also be metrically realized either by addition, i.e. 6/x, 4/x, etc., by division, i.e. as beat speeds: x/32, x/48, x/28, x/64, x/40, or by combination of both, i.e. 7/32, 6/48, etc. To every given number in the talea, as a bar, he assigns attack points (**Ex. 2.15**). These attack point patterns are used throughout the piece and later as variation on a bar, e.g. appearance of bar-spanning



Ex.2.15 [48]

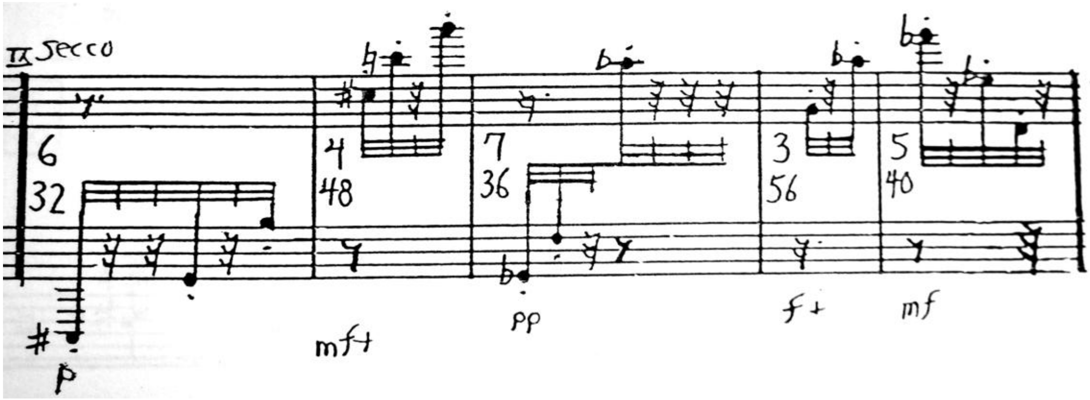
tuplets, which also retain theses attack point patterns, e.g. 7:3 tuplet will use the attack point pattern assign to the number 7 (**Ex. 2.15**) and so forth.[49] In this way all the material is realized in all the speeds and in all kinds of different impulse structures. Here we see this attack point pattern within the piece (**Ex. 2.16, 2.17, 2.18)**:

Ex.2.16



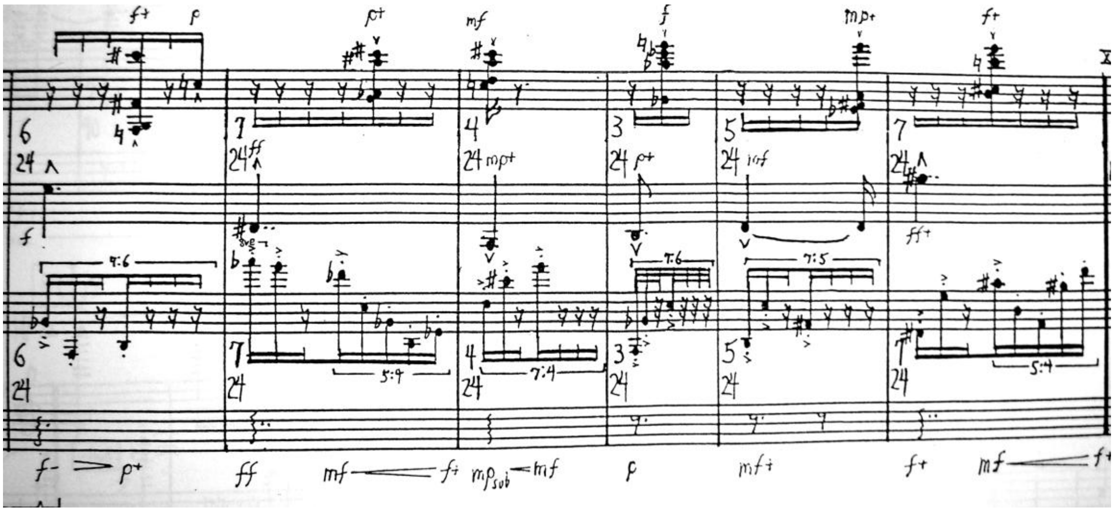
47 Cox 2004, p.93
48 Cox 2004, p.101
49 Cox 2004, p.100-104

In **Ex. 2.16** and **2.17** the talea is in its original order but the denominators are different. The same rhythm material is realized on another surface.



Ex. 2.17

Later in the piece this is developed further.



Ex.2.18

In **Ex. 2.18** we see (in the third note-string system) septuplets containing the attack point pattern of 7 and maintained in different ratios (7:4, 7:6, 7:5), while the denominator is fixed (24).

To avoid any misunderstanding about his working methods it is essential to understand the following:

> "…the taleas mentioned are not treated as fixed, quasi-Platonic serial proportions, but rather as energy shapes, each defined primarily by its contour (which can always be inverted) and made determinate by its constituent intervals, indicated by the numbers of the talea…"[42]

We see in this piece, *Doubles* for piano and tape, a clear relation to Ferneyhough's idea of recurring and transforming cycles, only it has been taken to a certain higher degree (or extreme), but he has a very different approach to the bar itself. We see in this piece much simpler material within the bars themselves than in Ferneyhough's or Mahnkopf's music, owing it to Cox's idea about the rhythmical action (and aesthetic view) of the downbeat. This brings us back to the term *density expression* which level Cox has clearly tuned down in order to focus on other complex processes but at the same time he allows us to recognize better the bars, and the material within, with the consistency of the attack-point-patterns. Despite all that, the music sounds extremely complex and is very related in aesthetics. We can say that his approach is different, but with similar results. In this piece we see irrational meter type **a, b** and **c**[50] stretched to its limit, i.e. up to bars constructed of 96^{th} notes (type a) and down to 1.5^{th} notes and 3.5^{th} notes. He sticks to these types even though the pianist has to perform the piece with a click track, which is not the case in the tempo scale piece (*5 kleine Lakunaritaten*) above by Mahnkopf (which has more difficult meter relationships, actually).

2.5 Other approaches

Other composers have of course made use of irrational meter in a completely different way, and simpler. All of the above composers (except Schnebel) can in some sense be associated with New Complexity, which is in no way a prerequisite for irrational meter usage.[51] Composers have used irrational bars as an object to "cut corners" in their music, for refreshment, and then only appearing at few places within the composition. Or, in a Stravinskian additive rhythmic way, adding to rhythms in different speeds (i.e. different from Stravinsky, who used only 8^{ths} and 16^{ths})[52], i.e. triplets-relations, quintuplets-relations etc., but usually you encounter only triplet speeds, i.e. irrational meter type **a** (simplest division: 12^{th} notes). The above ways are usually encountered within more free compositional methods, but also, of course, in highly calculated formal structural music.

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50 see chapter 2.2

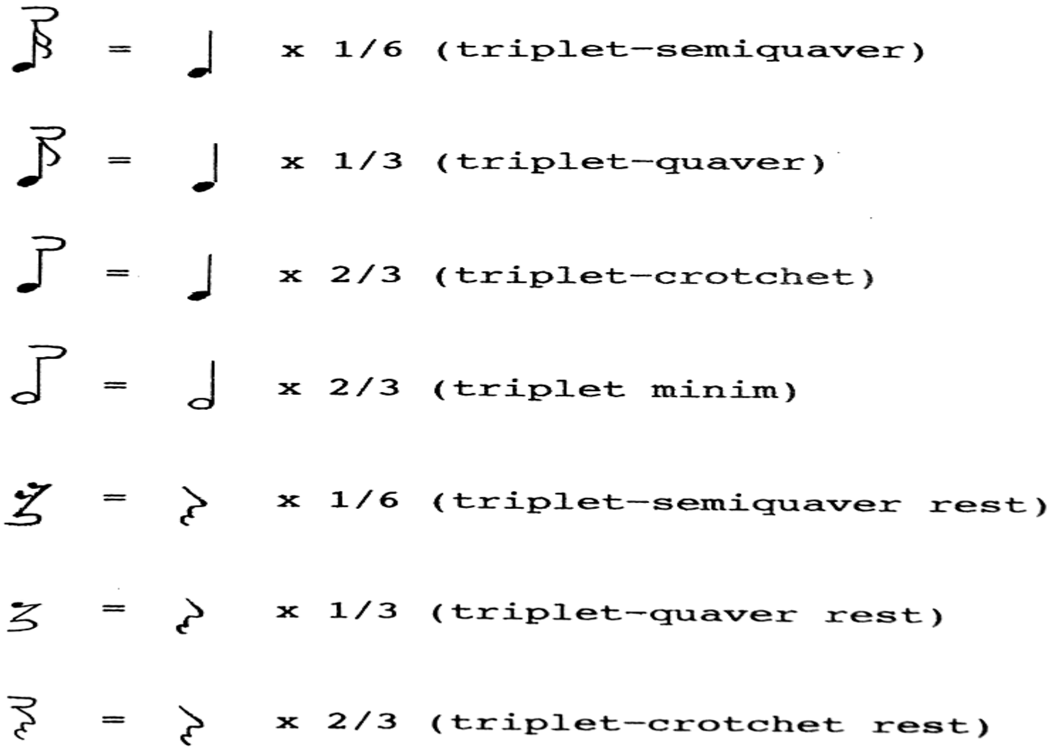
51 A multi-layered interplay of evolutionary processes occurring simultaneously within every dimension of the musical material (Grove Music (New Complexity)).

52 Le Sacre du Printemps is a good example of additive rhythms.

Thomas Adès is a composer who has made use of irrational meters in a quite different way than we have encountered so far. In one of his early pieces, *Catch op.4* for clarinet, violin, cello and piano, we see quite a stable usage of meter in general, i.e. long sections without changing the time signature at all.[53] As the piece progresses the music starts to move in triplets and soon thereafter we see usage of irrational meters: 4/12, 5/12, 1/12 and 1/6 bars are used as to cut the phrases a bit. This moment is a definite accent point within the piece since the music calms down a bit after this section (at least concerning meter changes). There we see that these irrational bars grow out of the triplets within the previous 2/4 bars, sort of in an additive stravinskian way. There, an irrational bar of 5/12 (or 4/12) has a clear rhythmical function, i.e. the pulse of the triplet is being used as the main element for addition, the accuracy of the pulse/meter change is important. Here, a bar is what it is (e.g. a triplet with one or two as an addition), not a durational space, or anything like it. This music has very different aesthetics from Ferneyhough or Mahnkopf, but more related, in some way, to Cox's preferences towards irrational meter, though Cox is far more complex and has always much more processes going on, which ensures that you never experience the bar as having a single rhythmical function plus that he constantly changes meter. Thomas Adès has used irrational meter in another ways as well. In his *Piano quintet* he explores all kinds of irrational meters types, in addition he also experiments with clashing of different speeds, i.e. irrational polymetric passages.[54] Also, one can find Ligeti like textures being altered with irrational meters. Always in his music the bar remains a bar with a rhythmical function, i.e. it doesn't become a conceptual formalized object with dense layers of impulse structures attached to it.

Jo Kondo is another composer who has made use of irrational meter thinking. Though, he doesn't really use irrational meters in a "conventional way". In his music you see time signature as series of notes, ¾ bar would be like ♩ + ♩ + ♩ etc., and he uses different note heads for triplet-related notes (type **a**), which he often attach to these sort of time signatures, giving them (the time-signatures) an irrational duration or slightly irregular sizes. In the performance notes to many of his pieces you'll see this:

[53] Ades 2002
[54] Fox 2004, p.52



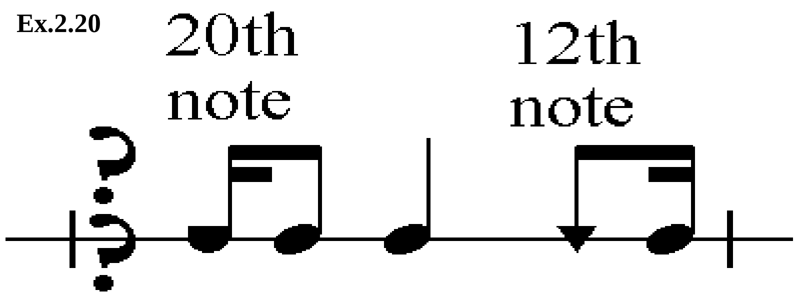
[55]

Ex.2.19

This brings us back to Henry Cowell who also wanted to and did use different note heads for these kinds of notes or note durations (**Ex. 2.19**). This gives, of course, a different kind of freedom within a bar than conventional irrational bars do, because within a 5/20 bar it remains quite complicated to write a single dotted 8th note, but with this notation you can put any kind of notes (any proportional relationship) anywhere, within any kind of meter. His scores, in the context of this thesis, poses the question:

How to call a bar, or what is the meter, which contains the following note durations: 20th note + 8th note + 4th note + 12th note + 16th note. This could look like this (**Ex.2.20**):

Ex.2.20



[55] Kondo 2000

This does not happen in Jo Kondo's scores, but nearly. One can easily expand this idea towards great complexity. This is a very different approach to the usage of different durational values and brings up different problems than "conventional" irrational meter use does, e.g. there is no establishment of any pulse if one would constantly change note values in this way (only the establishment of irregularity) (**Ex. 2.20**). It is, in some sense, diminishing the bar, the meter, to a single note, i.e. a note becomes a micro-bar. Or, perhaps it finally joins the mathematician use of the term irrationality, since here you truly enter the need for an *irrational* time signature.

3. Education

It should be understood for the following text that I use the word “speed” to indicate a pulse of a tuplet. This means that a speed of 5 means a quintuplet speed. The word “phrase” I use for a number of units with the same speed value, i.e. a phrase in 3 in the speed of five, means any three notes slurred together in a quintuplet speed.

3.1 The present situation and its transformation

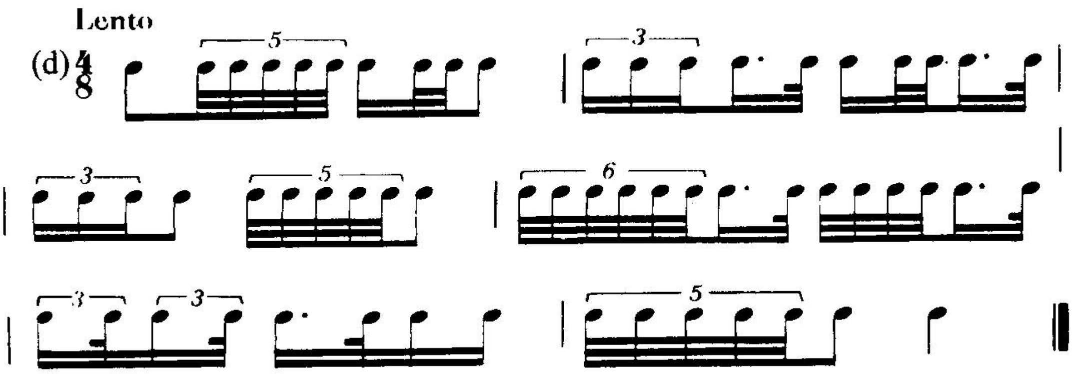
When one looks at the above compositions the speculation arrives about why irrational meter is not at all part of rhythmic education? More and more composers have started to use this phenomena in their compositions and the time will doubtless come that rhythm exercises will include 5/20 and 4/12 (at least) and alike bars. If one looks at the present rhythmic educational material one sees that all the preparation is already there, only we need just a little effort to cross the threshold and add this element to our material of rhythmic education.

Let us look at the important steps, which form the main part of rhythm education. Students learn first to establish fluency in rhythm through simple 4/4 exercises. With growing difficulty in rhythm, from 8th notes- to 16th notes- patterns, students encounter slowly different kinds of tuplets (little is done with phrasing in tuplets, more about that below). First there are triplets, and later on: quintuplets and septuplets. In addition to these, exercises in difficult meters follow and on towards a constant change of meter. This is the process in a nutshell. Of course it is not until the very end of this process that the student would be able to tussle the challenging exercises, which include irrational meters.

Let us look at examples on how we can transform rhythm exercises, which already exist. The book by Paul Hindemith, *Elementary training for musicians* originally published in 1946, includes a great deal of rhythm exercises from beginner to intermediate.[56] There, the focus is on rhythm exercises within one time signature or, with the more challenging exercises, two time signatures at the same time, i.e. a polymetric exercises. The polymetric exercises are hard to transform since they are intended to be performed by a single person, but they could be transformed in such a way that they become for two persons. The mono-metric exercises, which include

[56] Hindemith 1949

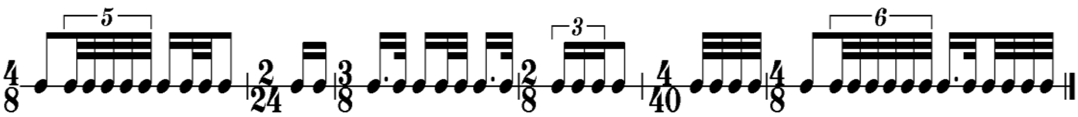
occasional triplets and quintuplets, are a splendid material for transformation. There one can take or add one or two from the tuplets to create irrational bars.
This exercise (**Ex.3.1**):



[57]

Ex. 3.1

could be transformed into this exercise (**Ex. 3.2**):


[58]

Ex.3.2

In **Ex. 3.2** I removed the last note from the triplet in the second bar of **Ex. 3.1** in order to create a 2/24 bar. We see the same procedure for the quintuplet in bar 3 of **Ex. 3.1**, for the creation of a 4/40 bar (**Ex. 3.2** bar 5).

Robert Starer wrote the book *Rhythmic Training*, which includes great number of exercises where the meter is constantly changing.[59] Chapter XI would be a great chapter for transformation. There we find exercises which include meters like 5/8, 3/16, 7/8, etc. These bars can easily be kept precisely like they are, i.e. keep the phrases and rhythm patterns within the bars, only we change the meter, to be more precise we change the denominators. 5/8 would then become 5/12 or 5/15, 3/16 would become 3/18, 3/20, 3/24, 3/28 or 3/30 etc.

The course *Advanced rhythm* takes a further step in rhythm training than conventional rhythm educational material does, especially in regard to phrasings.[60]

57 Hiundemith 1949, p. 119
58 Einarsson 2008
59 Starer 1969
60 This course was developed, and is taught, at the Conservatorium van Amsterdam.

This course trains the musician to be able to phrase easily within any speed (3, 4, 5, 6, 7). This material is based on Indian Carnatic music theory, which has very advanced use of rhythm and rhythm theory. The exercises deal with phrasing in 3, 4, 5 and 6 in the speed of 7 and phrasing in 3, 4, 6 and 7 in the speed of 5 and so forth for all the speeds. It is important to realize that in those phrasing exercises the phrasings overlap the beat constantly, e.g. phrasing in 3 in the speed of 5 starting on the 4th or the 3rd note of the quintuplet. These kinds of exercises increase greatly the feeling for proportions between different speeds, but the course doesn't take the step to cut the phrases into irrational meters, the course is though a great preparation for it. The exercises within the course always have the goal to finish the phrases "on the beat", i.e. the underlying pulse is kept constant. This means that phrasing in 4, within the speed of 7, will take 4 beats to come back to the original beat (4*7=28 = 4 beats in the speed of 7). This helps greatly to play all kinds of polyrhythms and more complicated rhythms, but there, in this course or in imaginable following course, the step would be really easy to take since participants gain great control over tuplets phrasings. When you have full command over phrasing in 4 in the speed of 7, and such, one can easily transform that into a 4/28 bar and create therefore exercises in phrasing with irrational meters.

3.2 Proportional hearing/feeling for pulse

These sorts of exercises mentioned above would greatly help to create a better sense for proportional pulse relationships. An explanation of that term would be the following: a detailed recognition of note-durations (speeds) in relation (only) to each other.[61] The above exercises, on the other hand, include already phrases, which would be better skipped in the early and the first steps towards irrational meter practice, but would be very important in the later stages. To create within a musician a strong feeling for different proportional pulse relationships we have to educate pulse in the same manner as we do with normal intervals, i.e. pitch intervals. There is no difference between pitch intervals and pulse intervals, only there is a difference of speed (frequency), this we learnt from Henry Cowell: the same proportions between the notes of a chord can easily be established as a pulse or rhythm since pitch is pulse

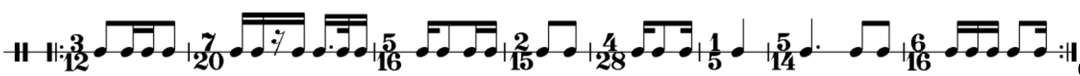
61 Einarsson 2008

at a higher frequency.[62] Since we humans are able to train ourselves to a high degree in recognising intervals and chords we can easily do that with pulse and rhythm.
The first exercises would then be (like with pitch intervals) to recognise the most common proportional pulse intervals. We can say that those are the speeds of 3, 4, 5, 6 and 7 (irrational meters: 12, 16, 20, 24, 28). What is important to understand for establishing a perfect or higher sense for proportional pulse relationships is that in these exercises, which help with proportional recognition, the feeling for 1 must be neglected, i.e. the feeling that a quintuplet means fitting five notes over the 1 must come to an end. The “1” here has to be a stretchable concept. For this to be established the following exercise, **Ex. 3.3**, would be greatly helpful.
Pulse intervals 3:4 and 3:5 (same as, so called, 4th and 6th in pitch intervals):


[63]

Ex.3.3

This can eventually be extended by increasing the numerators and by a transposition to all different speeds, 3-7, 4-7, 5-7, 4-6, etc., and adding more speeds (3-4-5, 3-5-7, etc.). Later in this process more challenging proportions (9, 10, 11, 12, 13, etc.) can be exercised and, finally, exercises which include phrasings and rhythm patterns (e.g. *advanced rhythm course* <u>add-ons</u> and *Starer* transformations, see above).
It is fully realistic that these sorts of exercises (**Ex. 3.4)** should be reached within a serious irrational meter training program.[64]


[65]

Ex.3.4

62 Cowell 1930
63 Einarsson 2008
64 It might take decades or a century, but it’s realistic nonetheless.
65 Einarsson 2008

I have, in this chapter, chosen to explore the method of adding chronologically to the existing material of rhythm education, i.e. a material which should/must be taught on an advanced level. It should be mentioned that other methods should be successful as well. E.g. one can imagine to introduce these meters at a very early stage in the rhythm education, along with the first instance of a triplet. It is obvious that above exercises mentioned within chapter **3.1** would, in that case, not apply. However, the exercises in chapter **3.2** might be an interesting material for beginners.

It is worthwhile to mention that I underwent a small research to see if there were anywhere courses or training programs, especially for irrational meter, being taught in the world today. That search resulted in no such place.[66]

[66] Questions were sent to Universities and Conservatories in England, The Netherlands, Germany, Austria and USA.

4. Conclusion

Henry Cowell in the early 20th century obviously innovated something. His ideas about rhythm created a vibration, which actuated certain type of thinking about rhythm and rhythm/metric structures. We have seen, in this paper, quite different results that can in one way or another be traced back to his ideas. Dieter Schnebel is perhaps the first one who tangled the problem Cowell set forth and found different solution from Cowell in regard to new durational notes, note heads and notation in general for irrational meters, mainly that he didn’t use different note heads, but an adaptation. The metric structures created by Schnebel triggered Brian Ferneyhough’s imagination. Ferneyhough is the one who took this phenomenon to a different level. Ferneyhough’s main concern and use of irrational meter is of course very related to his aesthetic view in general and in particular on the measure, the bar, i.e. on the durational space function of the bar, which he structures within his main metric structures as grids for his other compositional material (pitch material, impulses groups etc.) to “press through”. He makes use of irrational meter in his very own way and it’s very *purely* adjusted to his compositional technique. Ferneyhough influenced, as seen, many of his students and we looked at two of them who went even further than Ferneyhough in exploring irrational bars and structures. They make use of irrational meter while preserving their aesthetic view, though one can see a relationship with Ferneyhough’s usage and aesthetics. In Mahnkopf we saw two different ideas, which in some sense are related to tonality thinking. In *Rhizom*, Mahnkopf makes the meter into the subject of tempo-chromaticism, i.e. he slowly creates layers of individual voices, which then are chromatically altered in tempo through the use of irrational meters, which gives the same preciseness as pitch chromaticism, i.e. the proportional relations can be altered and tuned in a precise way. We also saw, in Mahnkopf, an example of constant usage, i.e. ever changing, of irrational meters through a certain tempo scale (*5 kleine Lakunaritaten*). There, we saw some thought relationship with Indian Carnatic music (jathi relationships), in regard to connections of bars (bars as phrases). With the use of a tempo scale we see the ideas of Henry Cowell being explored 60-70 years later by Mahnkopf. Frank Cox went even further with his “continuous usage” of irrational meter. He bases his compositions purely on irrational meters, which he makes use of to create his “non regular rhythm language". Cox disagrees totally, and aesthetically, with the idea that a bar is a durational space. He wants the bar and the meter to maintain their original meaning, i.e. the heaviness of the downbeat should have a rhythmic function. In regard to this idea we see less complex material within Cox's bars than from Mahnkopf and Ferneyhough but nonetheless his music gives the impression of high complexity.

The non-complexity composers have their own way of using irrational meter. They use them in a rhythmical way, i.e. the aesthetics are more straight forward, or less abstract in some sense. The pulse of a meter (or a bar) has a drive on its own, there are no layers of material. This we can see in Thomas Adès and in Jo Kondo. Kondo has though a different idea. He isn't using irrational *meter* per se but irrational note durational values in a free way. His meters are row of note-values, therefore, in a way, he makes even more complexity than the complexity composers because his bars can't even have a rational nor irrational meter attached to them. His music questions meter altogether.

It is certainly the main conclusion of this research that irrational meter is a vibrant phenomenon, which has been explored mostly, though, within the "complexity school" and remains considerably theirs. But other composers have taken up usage as well and applied it to their structures, and this is doubtless growing. Irrational meter is an element, a compositional material, which is being explored within the contemporary music field in many different ways and will for sure continue to do so. We have seen different solutions or methods in usage and different why's, or underlying reasons for usage. This could be summarised as the following:

Schnebel	free exploration, weight to single note values
Ferneyhough	durational space, proportions, transformations
Mahnkopf	tempo/meter chromaticism
Cox	rhythm function, continuous usage (complex results)
Adès	rhythm function, occasional usage (simple results)
Kondo	rhythm function (complex & simple results)

The problem of performance in regard to irrational meter may be traced back to education. In this thesis I tried to suggest ways, and material, to expand our rhythm educational material. There we saw that only a minor addition to our system of rhythm education is needed to ensure understanding and performance fluency of irrational meters. This would for sure encourage composers and performers further in this direction and create new music and better performance.

Irrational meter, when used to a certain degree, has a problem of coordination when used for an ensemble (also an educational issue). This is, of course, a known fact in general: that you can’t write as complicated music for an orchestra as for a solo. The complexity and difficulty of the score by Frank Cox for instance, *Doubles*, could never be performed by more than one person (and that is hardly even possible). There are pieces, though, which are not solo piece that use irrational meter, e.g. *Etudes Trancendantales* by Ferneyhough for oboe, soprano, cello and harpsichord is a good example, there we see an overall usage of irrational meter (the pieces mentioned by Adès and Kondo are also all chamber music).

The phenomenon of irrational meter usage has been much more explored within the solo repertoire and was taken as the main issue of this research, but it raises the question of ensemble and orchestra use of this phenomenon. This research doesn’t cover those scores, for ensembles and orchestra, which make use of irrational meter, but that would be an interesting study indeed.

Appendix A

Fabric

Explanation of New Rhythms and Notes

In musical time a whole-note ( ) is the unit by which all shorter time-values are measured, for instance an eighth-note ( ) is so called because it occupies one eighth the time of a whole-note; a quarter-note ( ) is so called because it occupies one quarter the time of a whole-note, etc.

The only regular system of subdividing a whole-note is by twos into halves, quarters, eighths,etc. If notes of other time values, for instance notes occupying one twelfth of a whole-note, are desired, they are called "eighth-note triplets" and written as eighth-notes, with a figure *3* over them, thus 3. Why not call them twelfth-notes, as would seem natural?

It is here proposed that all these irregular time-values be called by their correct names, according to the part of a whole-note they occupy. Thus 3 are third-notes instead of "half-note triplets" since each occupies one third of the time of a whole-note; 5 are fifth-notes instead of "quarter-note quintuplets, etc.

Although heretofore not suggested in notation, it will be seen that third, sixth, twelfth and twenty-fourth-notes form a related series; fifth, tenth and twentieth-notes another, and in fact, that a new series can be formed on each odd number and its divisions by two.

A new notation which brings out these relationships will be used as follows:

Whole-Note Series
Oval-shaped notes

Whole-note: half-note: quarter-note: 8th-note: 16th-note: 32nd-note:

Third-Note Series
Triangular-shaped notes

2-3rds-note: 3rd-note: 6th-note: 12th-note: 24th-note: 48th-note:

Fifth-Note Series
Square notes

4-5ths-note: 2-5ths-note: 5th-note: 10th-note: 20th-note: 40th-note:

Seventh-Note Series
Diamond-shaped notes

4-7ths-note: 2-7ths-note: 7th-note: 14th-note: 28th-note: 56th-note:

Ninth-Note Series
Oblong notes

8-9ths-note: 4-9ths-note: 2-9ths-note: 9th-note: 18th-note: 36th-note:

Eleventh-Note Series
Oval notes with stroke

8-11ths-note: 4-11ths-note: 2-11ths-note: 11th-note: 22nd-note: 44th-note:

Thirteenth-Note Series
Triangular notes with stroke

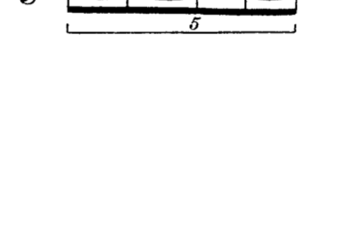
8-13ths-note: 4-13ths-note: 2-13ths-note: 13th-note: 26th-note: 52nd-note:

Fifteenth-Note Series
Square notes with stroke

8-15ths-note: 4-15ths-note: 2-15ths-note: 15th-note: 30th-note: 60th-note:

Following is part of the second measure of "Fabric", in both old and new notation. It will be seen that the new notation saves the use of several tied tones, as well as figures and brackets.

OLD



NEW

Appendix B

Interview with Brian Ferneyhough by Einar Torfi Einarsson - 2008

How did it first occurred to you to use, so called, irrational time-signatures (influences?)?

I think that the first score I saw which employed them was Schnebel's early pieces for string quartet. I found them useful in order to change the durational relationships between measures of already-composed material without changing the notation itself. (Carceri I).

What about the history, is there anyone else than Henry Cowell who could be called a pioneer in thinking in this direction?

Well, you could have a look at Wyshnegradsky, who tried to apply logarithmic values to individual rhythmic durations.

What function(s) does it have in your compositions? (e.g. Lemma-Icon-Epigram, Etudes Transcendantales)

Difficult question to answer. What is 'function'? The Third Quartet (2nd movement, towards the end) is a better example - you can actually see them working in 'temporal dissonance'.

What were the first pieces you experimented with irrational meters?

I THINK L=I=E was the first., but 1979-80 was a chaotic period of invention, so things got spread over several works.

What do you think about your students, i.e. Mahnkopf and Cox, who have *taken this feature of metric structure to its extreme? What about the future?*

I don't mind people using such things as a self-consistent aide-memoire to the inherently unstable time/rhythm dimension. If they can hear/play them, then fine with me. Cox has done a lot of work on the didactic transmission of complex materials. Conservatoires should have a solfège class dedicated to ratio-based rhythmic realization for performers - we know what a minor third is, but not a double-nested set of triplets. That said, it is of course up to the compositional context to validate usage, not vice versa.

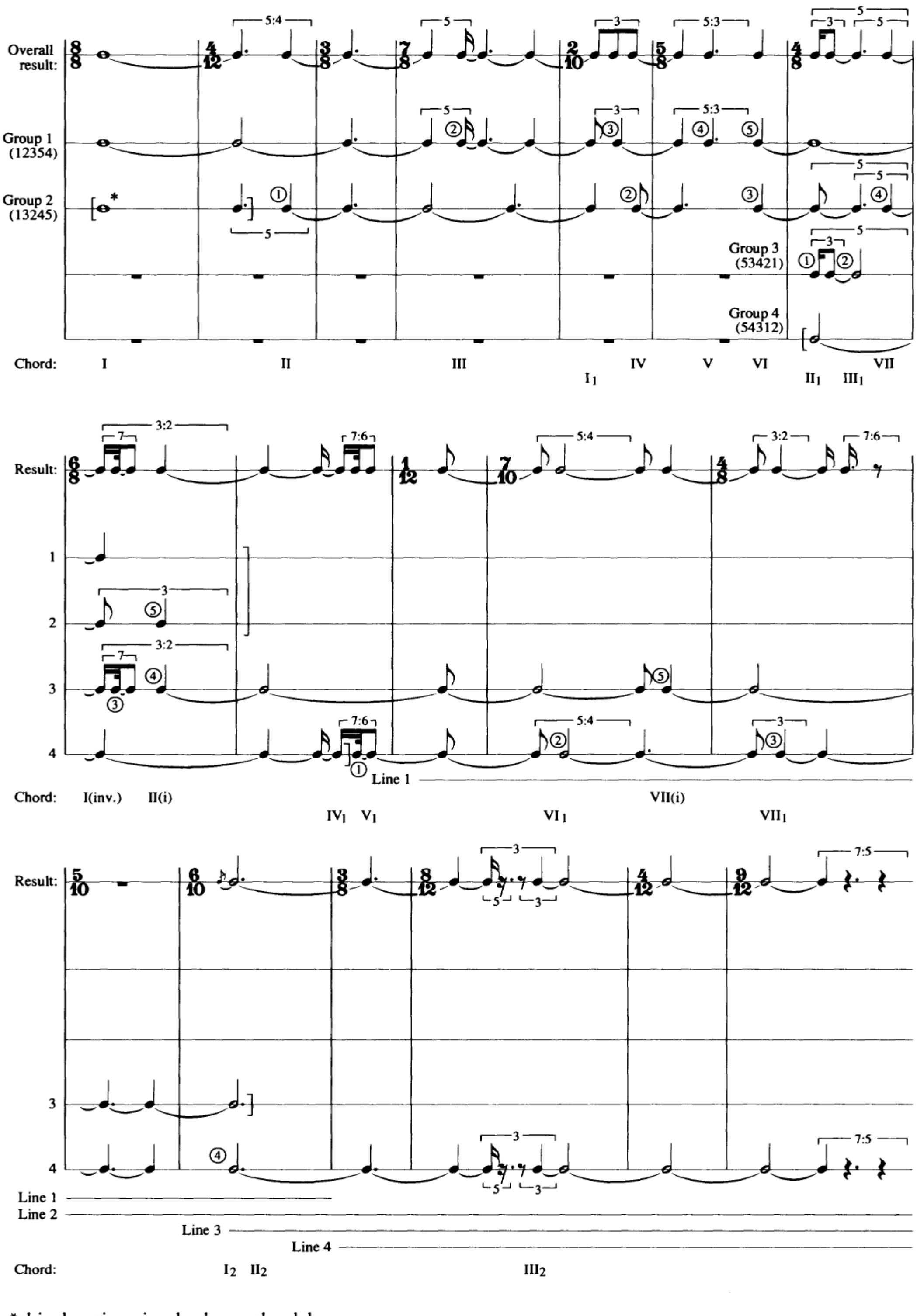
Are there any specific aesthetics linked with such metric structures? (as an empty, unformed, metrical space? Is there a visual imaginative element?)

There is probably a psychoacoustic component - i.e. what the performer senses/thinks/does when confronted with such structures. They are often employed to accommodate a principle of incompleteness (i.e. 4 out of 5 quintuplets) or to produce metric modulation correspondences where simpler values would not serve. I don't know if my own sense of 'metric space' as an independent value is employed by others. The 'dissonance' factor is important, as is the pointing-up of textures made of independently evolving density strata.

If you know any contrasting examples of irrational metre usage I would certainly be very pleased to receive any such information.

There are a few younger US composers like Aaron Cassidy (now working at Huddersfield University). You could try him. he is very articulate. Of course there was (is) Klaus K. Hübler, who still occasionally employs them in his otherwise rhythmically radically simplified recent compositions.Richard Barrett, of course has emploed them. I believe even such luminaries as Thomas Adès has been known to occasionally succumb. Chaya Czernowin as a relatively extensive praxis in such things.

Appendix C



*this duration simply shows the delay before the proper entry of the group.

(adapted from composer's sketches)

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